

## The Heuristic Approach to Problem-Solving And Students' Learning in Logic and Set Theory

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### ABSTRACT

*This study seeks to determine the effects of problem-solving heuristic approach towards students' learning in Logic and Set Theory. Furthermore, the study identifies the heuristic strategies used by the students; the extent of heuristic strategies in improving students' performance in terms of motivation and interest, knowledge acquisition, and attitude and behavior; the effects of heuristic approach in a student's ability in solving Logic and Set Theory problems in terms of pre-test and post-test result; and the significant difference between the pre-test and post-test of students under experimental groups. The study used quantitative as a method of research and descriptive research design where the respondents are chosen using purposive sampling. Survey questionnaires, and pre-test/post-test were administered as part of the study to gather necessary data from the respondents. The pretests were distributed to the thirty-eight (38) BSEd Mathematics students who are currently enrolled in Logic and Set Theory. After the two-week teaching, the post-test was administered. However, the survey questionnaires were distributed to all BSEd Mathematics students from first to fourth year in the above-mentioned school which was composed of 126 students in total.*

*The results were treated using scoring, weighted mean, ranking and z-test respectively. Findings have revealed that Heuristic strategies that are commonly used by the students are systematically listing and tabulating the data to solve Logic and Set Theory problems, reading the problem carefully and restating in their own words, and solving part of the problem by splitting complex problems into smaller parts. In addition, heuristics strategies contribution in improving the student's performance in Logic and Set Theory prior to the motivation and interest, knowledge acquisition, and attitude and behavior, have a general weighted average that is regarded as extent – hence perceived to have a satisfactory rating among the students. Also, this study showed that heuristic strategies had a significant impact on students' ability to solve problems in logic and set theory given the results of the experimental pre-tests were significantly better than the post-test. Lastly, it has been revealed that the 18.61 mean difference between the pre-test and post-test scores suggested that there was a significant difference between the academic achievement of students in Logic and Set Theory before and after the instruction integrating the four categories of Singaporean Mathematics heuristics namely representation, simplification, generic and pathway heuristics specified by Sage, T. R. (2022).*

**Keywords:** *Logic and Set Theory, Problem-Solving, Heuristic Strategies, Students' Learning*

### INTRODUCTION

Education is inevitably one of the most vital factors in developing the human resources that is crucial for a nation's progress on all fronts. In a broad sense, education refers to any activity that significantly influences how someone's cognitive, skill, and attitude are developed. It is made up of many disciplines that everyone should be taught and trained in. Mathematics is one of the oldest academic fields or disciplines that is taught in every school curriculum worldwide (Guinocor et al., 2020).

One of the required disciplines is mathematics, which teaches students how to think critically, systematically, rationally, and creatively. (Erna Yayuk et. al, 2020). The Trend International Mathematical Science Study Advanced (TIMMS) looked at trends in students' mathematical achievement and found that for more than 20 years, there have been some declines and no gains in the students' mathematical abilities.

In any educational system, teaching and understanding mathematics are considered as a significant concern. Guinocor et. al, (2020) claimed that mathematics is one of the most critical disciplines in the curriculum, that's why students are encouraged to study the subject. This perspective explains why children's math achievement guidance techniques are far more aggressive in the majority of Asian nations. In the Philippines, mathematics is a general education subject taught in primary and higher education. Students are expected to acquire and apply specific skills in this subject, including knowing and understanding, estimating, computing, and solving problems, visualizing and modeling, representing and communicating, conjecturing, reasoning, and making decisions (K to 12 Basic Education Curriculum). Since it is crucial, practically every field requires it as a subject. However, issues with mathematical proficiency are still present, not just in the Philippine context but also in other nations. In the 2018 Programme for

International Student Assessment (PISA), Filipino students were identified as one of those having the lowest performance levels among all the student participants across all participating nations. Less than 20% of students in mathematics showed the required level of ability (Level 2), and more than 50% displayed very low proficiency (below Level 1). These Filipino students have been clearly left behind their international classmates in terms of mathematics education as seen by their PISA scores, which were below the lowest level of competency. More than half of this age group of Filipino students also demonstrate weak mathematical ability. Between children in public and private schools, where the means were 343 and 395, there were substantial differences in the severity of low math performance (Department of Education 2019, 2019).

Teaching students to solve mathematical problems on their own is one of the key objectives of the school of mathematics. For Erif Ahdhianto et. al, (2020), teaching strategies and techniques are a significant component that promote student's academic achievement. Moreover, studying mathematics through activities allows learners to open doors for the development of their mathematical thinking, which is the cornerstone and the basis for a long-term sustainability in mathematics. In order to address these issues, the instructor can aid in students' learning by offering them strategies that allow them to think critically and solve mathematics problems at a high level.

Heuristic is an approach founded on the idea of experiential learning. As opposed to the conventional manner of information receiver or only being taught about things, this technique of teaching places students in the role of a discoverer (Singh et al. 2018). Decades later, academics popularized its usage where they elucidated heuristics as an approach used to solve unusual or non-routine math problems. In other words, heuristic strategies are viewed as the modus operandi for successful problem solving. It is the understanding of strategies, procedures, and approaches for solving problems with the objective of investigating the procedures and laws governing development and discovery.

Problem-solving is one the twin goals of mathematics education in accordance with the K to 12 Basic Education Curriculum. In accordance with this the CHED Memorandum Order no. 75 series of 2017, were issued allowing Higher Education Institutions offering the Bachelor of Secondary Education (BSEd) Program may exercise flexibility in their curricular offering. The order released specified all the curriculum components for each program under Article V, Section 9.1.

With this in line, by emphasizing their persistent re-use in numerous contexts, many concepts and techniques' broad applicability is shown. One of these contexts is the use of mathematical languages and reasoning in the application of problem solving in Logic and Set Theory. (Wolfgang, R. 2010).

Heuristic as a strategy can be applied in teaching logic and set theory class. In this setup, the students will be able to construct their own knowledge and create their own representation of the problem to solve it heuristically (Singh et. al 2018). In addition, using heuristics has the essential advantage of enabling students to draw appropriate conclusions without needing all the information or performing complex calculations. As a result of their inability to acquire or comprehend all the knowledge necessary to come to a fully logical conclusion, humans attempt to draw a conclusion that is at least acceptable or good enough with the knowledge they do possess.

This idea was guided by the belief that there are alternative ways in solving a mathematical problem. Based on this premise, the researchers decided to provide an investigation that will examine the effectiveness of utilizing heuristic approaches in solving mathematical problems specifically in logic and set theory. The four categories of Singaporean mathematics heuristic strategies specified by Sage, T. R. (2022), such as representative heuristics, simplification heuristics, pathway heuristics and generic heuristics will be utilized in the class in order to determine its significant impact on increased students' motivation and interest in mathematics, knowledge acquisition, and attitude and behavior.

## OBJECTIVES

This study aims to investigate the effects of the problem-solving heuristic approaches towards the learning of BSEd Mathematics Major Students of Batangas State University TNEU ARASOF Nasugbu in Logic and Set Theory.

Specifically, the study sought answers to the following questions:

1. What are the heuristic strategies used by the students in solving Logic and Set Theory problems?
2. To what extent does heuristics approach contribute to improving a student's performance in Logic and Set Theory in terms of:
  - 2.1 motivation and interest in Mathematics;
  - 2.2 knowledge acquisition; and
  - 2.3 attitude and behavior?

3. What are the effects of heuristic approach in a student's ability in solving Logic and Set Theory problems in terms of pre-test and post-test result?
4. Is there a significant difference between the pre-test and post-test results of students under the experimental groups?
5. What mathematics learning material applying the heuristic approach can be prepared by the proponents?

## MATERIALS AND METHODS

### Research Design

The researchers facilitated a quantitative research design. Specifically, it utilized a descriptive research design to assess whether the implementation of various problem-solving heuristic approaches in achieving students' learning was effective or not. This used survey questionnaires and pre-test and post-test as the main data gathering instruments. In quantitative research design, the study was focused on using descriptive approaches to gather reliable data and the use of statistical methods and tools to analyze and interpret the data gathered.

### Sampling Design and the Respondents

The main respondents of the study are the First Year College students of Bachelor of Secondary Education Major in Mathematics from Batangas State University - TNEU ARASOF Nasugbu. The experimentation was restricted only to them, as they are the only program under College of Teacher Education (CTE) who are currently taking the course Logic and set Theory.

The researchers utilized purposive sampling in choosing the subject of this study since the researchers used their own discretion when selecting individuals from the population to take part in their surveys.

### Data Gathering Instrument

The instruments used in this study to acquire the necessary information were survey questionnaires and pre-test and post-test. The questionnaire is composed of carefully prepared questions to get valuable information and data while the Pre-Test and Post-Test were constructed and then validated to assess the perceptions of students towards the significant differences in students' learning before and after the utilization of heuristic approaches of problem-solving in Logic and Set Theory.

### Statistical Treatment of Data

The data collected in the study was subject to various statistical treatments. The data was coded, tallied and tabulated for better presentation analysis and interpretation of the result. The statistical method includes:

**Frequency** presents the number of the actual responses of the respondents to a specific question item in the questionnaire.

**Weighted Mean** is used to determine the average responses of the research participants in the different parts of the survey questionnaire.

**Ranking** is used to assess the order or responses to the items from the need's assessment questionnaire with rank 1 as the highest.

**Z-test** the means of two groups are compared using this method. It is frequently employed in hypothesis testing to ascertain whether a procedure or treatment genuinely affects the population of interest or whether there is a meaningful link between two groups.

## RESULTS AND DISCUSSION

### Extent of Heuristic approach contribute to improving a student's performance in Logic and Set Theory in terms of Motivation and Interest in Mathematics

The results revealed that the Heuristic Approach of problem solving is to a great extent in improving the students' performance in Logic and Set Theory – mathematics subjects in terms of motivation and interest. It is clearly apparent as the majority of the respondents strongly agreed that they want to get good grades on tests, quizzes, assignments and projects which have an average of 3.89 and next to the statement, "*I am aware that I need mathematics in the future*" – having a weighted mean of 3.71. The idea of Heuristic Strategies role in improving students' performance was supported as well by the fact that First Year BSEd Mathematics students strongly agreed to the statements, "*I feel comfortable working on Mathematics problems*" which have a weighted mean of 3.60 and the statement "*I make myself prepared for the mathematics subject*", which had gained an average of 3.59 and the statement "*I am interested in doing Mathematics activities*" that gained a weighted mean of 3.53. The strong agreement of the students that they are motivated when they answer mathematics equations correctly is clearly evident as the majority of them favored to the claim with a weighted mean of 3.52 together with the statement that they are aware that Mathematics helps them to understand the importance of other subjects and also, they are interested in

learning Mathematics and having a passing grade which has all gained a weighted mean of 3.49. In connection to this, the respondents also show their utmost agreement to the statement that they listened attentively to the lecture of their Mathematics Professor that had a weighted mean of 3.45 and lastly, to the statement were their responses got the lowest rank and least weighted mean of 3.26 stating that they do agree that they participate in the discussion, answering exercises and clarifying things they do not understand. All in all, the students' responses clearly revealed that Heuristics strategies of problem solving are to a great extent in improving students' performance in terms of motivation and interest with a general weighted mean of 3.55, verbally interpreted as they strongly agreed.

These findings are relevant to the findings of the study entitled "*The Use of Problem-Solving Heuristics Approach in Enhancing STEM Students Development of Mathematical Thinking*" by Singh, Parmjit et. al (2022). The goal of the previous study was to evaluate how a problem-solving heuristic application technique affects students' growth in mathematical thinking. The mathematics curriculum for primary school places a strong emphasis on high-interest, career-related themes. Through engaging in activities that are mapped to the standards of the pertinent academic areas, students will explore cross-disciplinary topics, encourage creativity, and build higher order thinking skills. Through exploration activities, mathematical challenges, and real-world connections, students will apply what they have learned. Singh, Parmjit et. al (2022) study came up with the conclusion that a heuristic approach must be used in order to foster students' growth as mathematical thinkers thus it plays an important role in dealing with the students' motivation and interest to learn more about mathematics. With these relevant findings, the researchers therefore concluded that Heuristic strategies of problem-solving are of great extent in enhancing students' performance in Logic and Set Theory in terms of motivation and interest.

#### **Extent of heuristics approach effects in improving the student's performance in Logic and Set Theory in terms of Knowledge Acquisition**

The results expressed the strong agreement of the first-year students of BSEd Mathematics to the idea that Heuristic Strategies of problem solving is to a great extent in improving their performance in terms of knowledge acquisition. The tabulation revealed that with a weighted mean of 3.69, the students strongly agreed that they are aware that when solving Mathematical tasks, they need to know the correct procedure. The students showed their strong agreement to the idea that they know that Mathematics improves their thinking capacity and that they know the fact that it provides an understanding of the world around them, these two statements all got an average of 3.68. Garnering an average of 3.66, the students also strongly agreed that they know that Mathematics emphasizes argument, reasoning and logical analysis. Additionally, students also showed a strong agreement on the thought that they believe that Mathematical problems can be solved correctly in many ways with weighted mean of 3.63, and to the statement that they are aware that Mathematics deals with situations and ideas that come from the real world and they are familiar with Mathematics which requires much practice, correct application of non-routine and problem-solving strategies that gained 3.58 mean/average. Students also favored the idea that they are familiar with Mathematics as it is important to other courses that gained an average of 3.55. However, the students showed their disagreement to the statement concerning their beliefs that getting the right answer is more important than understanding why the answer works wherein their responses accumulated only 2.47 weighted mean that clearly tells that they do disagree with the statement. To sum up, students of BSEd Mathematics – first year totally strongly agreed that Heuristic Strategies of solving contribute to their academic performance in terms of knowledge acquisition having a general weighted mean of 3.51 that falls under the level of agreement – strongly agree.

These findings of the study are relevant to the study entitled "Impact of Heuristic Strategy on Students' Mathematics Ability in High Order Thinking" owned by Hardi Tambunan (2018). Both studies aimed to ascertain the impact of heuristic strategy on students' mathematical ability in high order thinking. Both the results showed that integrating Heuristic strategies as a method of solving affects students' abilities in (1) conceptual understanding, (2) creativity, (3) mathematical communication, (4) problem solving, and (5) reasoning. The results suggest that heuristic strategies have a very strong influence on students' mathematical competencies and have a significant impact on the mathematical aptitude of students with high levels of thought. Based on the findings, the researchers therefore conclude that both studies suggest that heuristic strategies are of great extent in improving students' performance in Logic and Set Theory in terms of knowledge acquisition.

#### **Extent of heuristics approach effects in improving the student's performance in Logic and Set Theory in terms of Attitude and Behavior**

The tabulation revealed that students strongly agreed that they enjoy doing problems when they know how to work the problems out. The statement garnered 3.71 weighted mean and a rank of 1. It was revealed as well that they strongly agree to the fact that they are proud to say that Mathematics is their favorite, and they like it better than any other subject which got a 3.51 average. Additionally, students agreed as well that they think they are good at Math. This statement was found to have a 3.45 average. Students also agreed to the statements — they are eager to participate in discussions that involve Mathematics problems garnering a mean of 3.44, and to the idea that they don't believe that Mathematics is boring with a mean of 3.42. Revealing a 3.38 average, students expressed that they do like mathematics but they like other subjects the same. Also, students believed that they have confidence when taking a Mathematics problem, gaining a 3.37 average. Also, students showed their agreement in the idea that they enjoy the challenge of hard Mathematics problems and to the statement that they are aware that learning Mathematics is not frustrating that resulted in 3.21 accumulated weighted mean. However, the first-year students expressed their disagreement with the statement "*I believe that Mathematics is not confusing*" with a weighted mean of 2.37 and a rank of 10. To sum up, students from BSEd Mathematics first year at BatStateU TNEU ARASOF-Nasugbu clearly showed their agreement that problem-solving heuristic strategies are indeed to a great extent in improving students' academic performance in terms of attitude and behavior as the overall mean responses resulted to 3.31 which falls under the level of agreement — agree.

These results were relevant to the product of the study entitled "*Challenges of Mathematics Learning using Heuristic Strategies*" by Pramita & Rusmayadi, (2018) wherein the previous research suggested Heuristic learning techniques help students understand concepts and solve problems more quickly, which directly enhances learning results and that in addition to the cognitive elements, pupils' self-efficacy can also be increased. Self-efficacy refers to an individual's belief in his or her capacity to execute behaviors necessary to produce specific performance attainments. With these findings, the researchers therefore concluded that relevancy exist between the results of the present study and the previous study of Pramita & Rusmayadi, (2018) and that both studies suggest that Heuristic strategies are of great importance in improving students' performance in Logic and Set Theory in terms of attitude and behavior.

#### **Mean of Individual Pre-test, Post-test of First Year BSEd Mathematics Students**

The pre-test of an experimental group gives a total mean of 21.47. After a two-week teaching period, the researchers administered another set of exams which were the post-test. Upon checking their post-test examination, the researchers found out that students' performance in Logic and Set Theory had increased significantly with the mean of 40.08 in their post-test. Comparing the mean results of the pre-test and post-test, with the mean of 21.47 and 40.08, it is identified that students performed well in their posttest as it gives a mean difference of 18.61.

The findings suggested that the experimental group who have received a treatment wherein various heuristic strategies of problem-solving were introduced to them, significantly improved their performance. All in all, the result showed that there was a significant increase in their performance in Logic and Set Theory class which was clearly apparent in the mean difference of the students' pre-test and post-test scores.

The results of the findings expressed that the mean difference of the general mean of the experimental group tests scores was 18.61 indicating that the application of heuristic strategies in solving logical problems was effective.

The findings crafted by the researchers are relevant to the previous study conducted by Singh, Parmjit et. al (2022), in their research entitled "*The Use of Problem-Solving Heuristics Approach in Enhancing STEM Students Development of Mathematical Thinking*". Both the previous and the present study aimed to evaluate how a problem-solving heuristic application technique affects students' growth in mathematical thinking. Both were a two-phase study that analyzed and gathered the data using pre-test and post-test results. Singh, Parmjit et. al (2022) study and proponents' both look at the success of heuristics as a strategy. Like the result of the previous study, in the test of mathematical thinking they've done, they found out that the students who received the heuristic application treatment outperformed the students who did not receive the treatment. Therefore, the researchers came up with the conclusion that a heuristic approach of problem-solving must be used in order to foster students' growth as mathematical thinkers same as in the present study wherein based on the analyzed findings, solving problems heuristically as an strategy is recommended to improve first year students' performance in Logic and Set Theory class.

#### **Output for Independent Z-test based on the Pre-test and Post-test Results of Experimental Group**

The thirty-eight (38) students who have received experimental treatment in their class acquired an increased performance (Pre-test = 21.47) (Post-test = 40.08) after they undergo on two-week instruction utilizing various problem-solving heuristic approaches. This students' performance improvement was significant since based on the findings revealed, the researchers failed to accept the null hypothesis for  $t = 8.04$   $p(0.000881) < 0.05$ . Simply, the

class after receiving experimental treatment performed better in their Logic and Set Theory class than when they are taking a conventional method of instruction. The findings also revealed the large difference between the mean of both the pre-test and post-test scores of experimental groups. The 18.61 mean score difference clearly explained that students' increased class achievement. Since the z-value ( $z=8.04$ ) was higher than the critical value ( $CV=1.96$ ) proponents claimed that the problem-solving heuristic approach implemented during the instruction with the experimental group is an effective strategy to solve logical problems easily. Therefore, results were statistically significant, and the proponents rejected the null hypothesis.

Findings of the study were similar to the results of an experimental study conducted by Abraham D. Cacay (2022). Both researchers used pretest and posttest and both studies viewed heuristic strategy as an effective approach to the problem-solving process. In Cacay (2022) previous study, the mean percentage scores of the post-test results showed an improvement in the respondents' learning performance and that the strategy encourages student retention as seen by the rise in the delayed post-test mean similar to the results of the researcher's current study. The study also revealed that there is a considerable distinction before and after using the innovative strategy, same as the findings in the current study where it is proven that there's a significant difference between the pre-test and post test results of students under the experimental groups.

## CONCLUSIONS

Based on the above-stated summary of the findings, the following conclusions were crafted:

1. The researchers concluded that the most used heuristic strategies are (1) systematically listing and tabulating the data to solve Logic and Set Theory problems, (2) reading the problem carefully and restating it in their own words, and (3) solving part of the problem by splitting complex problems into smaller parts so, students must use the above-mentioned heuristic strategies in problem-solving to acquire a holistic package towards their mathematical thinking development. However, the least used heuristic strategy was looking at the end results and working backward towards the beginning.
2. Students are aware that they need mathematics in the future and that they aim to get good grades in tests and quizzes in their Logic and Set Theory class, thus the utilization of problem-solving heuristic approaches allows them to feel motivated and interested.
3. Heuristics improved students' thinking capacity specifically in making an argument, reasoning and logical analysis thus these approaches were significant in nurturing learners' means to knowledge acquisition.
4. Heuristics strategies provide students with confidence and positivity in answering logical problems particularly when they know how to work out the solutions. Thus, the study viewed the strategies as a means to cultivate students' attitude and behavior in Logic and Set Theory class.
5. Revealed in the study that Heuristics had a significant impact on students' ability to solve problems in logic and set theory given the results of the experimental groups' post-tests were significantly better than their pre-tests scores.
6. The comparison of the scores of the students from the two different sets of examinations had revealed a significant relationship. Thus, the problem-solving heuristic approaches implemented during the instruction with the experimental group is an effective strategy to solve logical problems easily.

## RECOMMENDATIONS

From the results and assumptions of the study, the following recommendations were recommended:

1. Researchers recommend that teachers should introduce problem-solving heuristic approaches to students so they could use it as a means to enhance their learning in Logic and Set Theory class.
2. Researchers recommend that teachers should use a developed learning plan for Logic and Set Theory courses with the integration of heuristic strategies to further address the learning challenges in a class instruction.
3. Teachers should practice applying heuristic approaches to their innovative teaching strategies in order to help students improve their performance in the class.
4. Future researchers may use this study as an additional and supplementary reference to serve as a foundation for the development of new research related to the topic.
5. For further validation, future researchers may work on similar studies investigating the effects of problem-solving heuristic approaches towards students' learning with a larger sample of responses.

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